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Vulnerability of Approximate Targets

Aivars Celmiņš

ARL-TR-154 June 1993

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This report describes the computation of vulnerability in case of approximately specified targets and threats whereby the specifications are assumed to be vague but not stochastic. Such specifications are typical for hypothetical or generic weapon systems. Treating the inaccuracies within the framework of possibility theory we obtain two confidence measures for the occurrence of a kill, namely, the values of possibility and necessity of kill. These measures replace the usual probability of kill that is an appropriate measure in cases of stochastic inaccuracies. Possibilistic vulnerability analyses can be particularly useful for the early development phases of weapon systems.

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PREFACE

Vulnerability estimates often are needed for approximately known targets and threats, for instance, when hypothetical or generic weapons are considered in the early phases of the development of a weapon system. A probabilistic treatment of inaccuracies of the descriptions of target and threat is proper only if they are stochastic, whereas non-stochastic inaccuracies are better treated by possibility theory that has been specifically developed to handle such uncertainties. This report describes the application of possibilistic concepts and techniques to the estimation of vulnerability. The results of a possibility analysis are values of two confidence measures called the possibility and the necessity of kill, instead of the more common probability of kill that is a proper measure if the uncertainties are stochastic. The possibility measure can assume values between zero and unity, corresponding to impossible and entirely possible occurrence of kill, respectively. The necessity measure also varies between zero and unity and expresses the certainty that a kill takes place.

This report has two purposes. First, it gives an outline of the theoretical aspects of possibilistic vulnerability analyses. Second, it tests the practicality of the approach: it is conceivable that the theoretically sound approach produces meaningless results, for instance, if a small uncertainty in the system description typically produces a possibility of kill that equals unity and a necessity that equals zero. Therefore, we present the numerical vulnerability analysis of a hypothetical tank in addition to the description of the theory. For that analysis, we defined a "typical" tank in the sense that all numerical specifications of its size, shape, armor, and response to attack were within realistic orders of magnitude. We then added liberal uncertainties to some input data and calculated the two confidence measures. The results show that the possibilistic approach is viable.

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1. INTRODUCTION

Vulnerability analysis is concerned with quantitative characterizations of the results from a pairing of a threat (projectile) with a target. A traditional characterization is the set

$$V = \{ L_{/H} , P_{K/H} , P_H \} \ , \tag{1}$$

where $L_{/H}$ denotes the losses of functions of the target given a hit, $P_{K/H}$ the probabilities of kill given a hit, and P_H the probability of hit. $L_{/H}$ is a vector where each component corresponds to the loss of some specific function, e.g., mobility, target acquisition, etc. Similarly, $P_{K/H}$ is a vector with at least three components corresponding to mobility, firepower, and catastrophic kills, respectively.

V characterizes the vulnerability of the target for each encounter, that is, for each impact point on the surface of the target. A simpler general characterization of vulnerability is obtained by averaging the elements of V over the target surface. The type of averaging depends on the purpose of the analysis. An often used average is, for example, the "sight average" where the averaging is done over a silhouette of the target. Alternatively, one might calculate weighted averages, taking into account the aim point and dispersion of the attacking weapon, or computing the weights according to an assumed distribution of attack directions, or using some other weight distribution.

Values of the components of $P_{K/H}$ are obtained as follows. The target is broken down into a number of elements that are sufficiently small so that their state after a hit can be reasonably predicted, but not too small so that their interconnections with other parts of the target can be represented in a fault tree of reasonable size. The state of the target after an attack is obtained by tracing the trajectory of the projectile through the target and noting which elements have been hit. More sophisticated calculations also include models of projectile breakup and spall generation behind the armor. In such calculations, one also traces the trajectories of fragments and spall particles. An analysis of the state of the affected elements and of the corresponding fault trees provides the values of the $P_{K/H}$ for the particular attack. We then repeat the procedure for other impact points and finally, average over the target. The values of the components of the loss-of-functions vector $L_{/H}$ are obtained by a similar process.

An important part of the described process is the determination of the state of a target element after a hit. This can be done with the help of decision functions $C(e,t)$,

where e is a parameter vector describing the element and t is a parameter vector describing the threat. An element is assumed killed if $C(e,t)$ is positive, that is, we assume the following

$$\begin{aligned} C(e,t) \geq 0 &\rightarrow \text{element killed} \ , \\ C(e,t) < 0 &\rightarrow \text{element intact} \ . \end{aligned} \tag{2}$$

A partial kill, i.e., a non lethal damage usually is not considered at the element level. In terms of a binary state function, the state of the element after the pairing of e and t is

$$S^{(1)}(e,t) = \begin{cases} S^{(0)}(e) & \text{if} \quad C(e,t) < 0 \ , \\ 0 & \text{if} \quad C(e,t) \geq 0 \ , \end{cases} \tag{3}$$

where $S^{(0)}(e)$ and $S^{(1)}(e,t)$ are the values of the state function before and after the encounter, respectively.

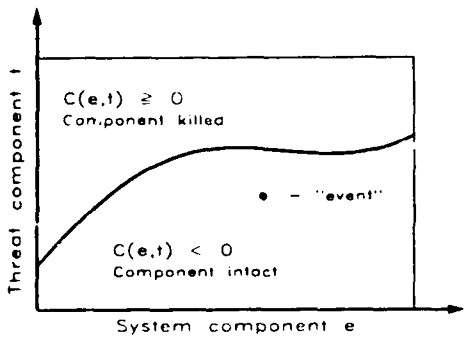

Figure 1. Damage criterion.

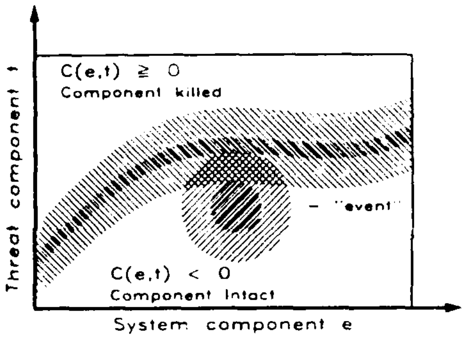

Figure 2. Fuzzy damage criterion.

The decision equation $C(e,t) \geq 0$ defines in the event space (e,t) a set of events that correspond to a kill of the elements e (see Figure 1). One can determine if an event corresponds to a kill by matching the given event with an element of this set, and in case of a match assume that the element e is killed. In practice, one has to deal with pattern matching in an uncertain environment. The uncertainties can be either stochastic or non-stochastic. In the stochastic case, one assigns a predetermined probability of kill to the element in case of a perfect match and assigns zero probability of kill if $C(e,t)<0$. Non-stochastic uncertainties are modeled by assuming that the decision functions $C(e,t)$ and their arguments are only approximately known (see Figure 2). Then one considers partial matches for events that are close to the boundary of the kill set and assigns confidence measures to the match or kill in accordance with assumed possibility distributions of the uncertain input. This approach is attractive because it allows one to model explicitly the approximate knowledge of the decision function $C(e,t)$ and of the event parameters e and t. The theory that treats such

partial matchings is possibility theory. It provides two confidence measures for the matching that have the meaning of a possibility and a necessity, respectively, that the target has been destroyed.

The next two sections give a short outline of the basic concepts of possibility theory and fuzzy sets. (The latter are needed for the implementation of the theory.) The rest of the report describes the application of the concepts to vulnerability problems. Section 7 presents a numerical example that was constructed to test the feasibility of a possibilistic vulnerability analysis.

2. CONFIDENCE MEASURES

Possibility theory (Zadeh 1978, Dubois and Prade 1988) deals with measures of confidence of events. Particularly interesting for our purposes are two confidence measures called possibility and necessity. Both measures map the event space to the unit interval. The possibility $\Pi(A)$ of an event A is zero if A is not possible and equals unity if A is certainly possible. The necessity N is defined by $N(A) = 1 - \Pi(\neg A)$, where $\neg$ denotes negation. $\Pi(A)$ expresses to what extent A is possibly true and $N(A)$ to what extent A is certainly true.

As an example, consider the caliber of the main cannon of a future tank. From additional information, such as the approximate size of the tank, we may know that any caliber between 20 mm and 350 mm is technically possible, though not to the same extent. Therefore, we may assign to the possibility measure $\Pi_T(c)$ of "tank cannon" positive values for $c \in [20,350]$, but assign $\Pi_T(c) = 1$ only in a smaller interval, for example, in $[20,150]$. We shall come back to this example later and show how the possibility distribution is modified when additional information becomes available. Note that this approximate description of a future caliber is quite different from a probability distribution. Depending on the available information and goal of the analysis, either a possibilistic or a probabilistic description might be preferable. In this report, we are concerned with the former.

The calculus of possibilities is different from the calculus of probabilities. The possibility and the necessity of a conjunction are, respectively,

$$\left.\begin{aligned} \Pi(A \wedge B) &= \min \{ \Pi(A) , \Pi(B) \} , \\ N(A \wedge B) &= \min \{ N(A) , N(B) \} , \end{aligned}\right\} \tag{4}$$

whereas a disjunction is calculated by

$$\left.\begin{aligned} \Pi(A \vee B) &= \max \{ \Pi(A) , \Pi(B) \} , \\ N(A \vee B) &= \max \{ N(A) , N(B) \} . \end{aligned}\right\} \tag{5}$$

In comparison, the corresponding formulas in probability theory are

$$P(A \wedge B) = P(A) \cdot P(B) \ , \tag{6}$$

and

$$P(A \vee B) = 1 - (1 - P(A)\,) \cdot (1 - P(B)\,) \ . \tag{7}$$

There are other differences. The most striking one involves the contrary event $\neg A$. The probability $P(A)$ completely determines the probability of the contrary event: $P(\neg A) = 1 - P(A)$. In possibility theory, the corresponding relation is different because the possibility of an event does not exclude the possibility of the contrary event. Consequently, one has only the weaker relations $\Pi(A) + \Pi(\neg A) \geq 1$ and $\mathrm{N}(A) + \mathrm{N}(\neg A) \leq 1$.

The calculus of possibility distributions is identical to the calculus of fuzzy sets and, therefore, possibility distributions can be considered as fuzzy sets.

3. FUZZY SETS AND FUZZY DESCRIPTIONS OF WEAPONS

A fuzzy set $\tilde{X}$ is defined by its membership function μ_X that maps the universe of discourse to the unit interval. The concept of a membership function is a generalization of the characteristic function of an ordinary (crisp) set. In case of ordinary sets, an element either is or is not a member of the set, depending on the value of the characteristic function that assumes either the value unity or zero. In case of fuzzy sets, the membership function indicates a degree of membership in the set. Figure 3 shows the membership function $\mu_X(g)$ of a fuzzy set $\tilde{X}$ where the universe of discourse is the event coordinate g. The set $\tilde{X}$ consists of all those elements of g that have a positive membership value. The fuzzy set $\tilde{X}$ may represent, for instance, the possibility $\pi(g)$ of the event g. Then the value $\mu_X(g) = \pi(g)$ expresses to what extent g is possible.

A fuzzy set is called normalized if the supremum of its membership function equals unity. The subset where the membership function equals unity is called the core of the fuzzy set. The set where $\mu > 0$ is called the support of the set. Mathematical operations with fuzzy sets have the flavor of interval arithmetic but are, of course, different.

We return to the example of the main cannon of a future tank and assume that the main weapon has a large caliber. One can define the concept "large caliber cannon" by a fuzzy set with a membership function that represents the possibility distribution $\Pi_L(c)$ of such weapons. For instance, one might assign zero values for $c \leq 70$ mm and let $\Pi_L(c)$ increase to unity as c increases to 130 mm. We then obtain the possibility distribution Π_M of the main weapon's caliber of the future tank with a large caliber weapon by a conjunctive combination of the distributions for "tank cannon", Π_T, and

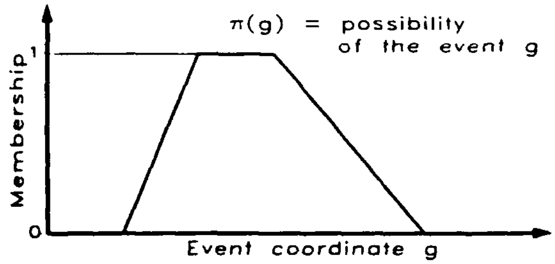

Figure 3. Possibility distribution.

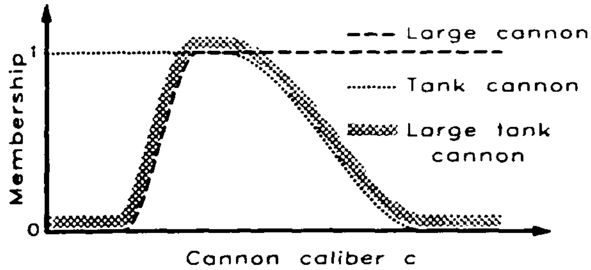

Figure 4. Combined possibility.

"large cannon", Π_L, (see Eq. (4) and Figure 4):

$$\Pi_M(c) = \Pi_{T \wedge L} = \min \{ \Pi_T(c) , \Pi_L(c) \} \quad . \tag{8}$$

Other attributes of the tank such as the various armor thicknesses and types, the engine and the shape of the vehicle can be similarly specified in terms of possibility distributions. This possibilistically described tank may be paired with known (crisp) threat ammunition or with approximately defined future rounds. The corresponding algorithms are described in the following sections.

Uncertainties in the specifications of existing weapon systems can be handled in principle in the same manner. Such uncertainties might involve, for instance, cable connections that can differ between specimens, or other detail that can vary between lots. Sometimes such variations can be treated in a probabilistic manner if detailed descriptions of the target and its variations are available. However, even in such cases a possibilistic treatment can be preferable, because it allows one to avoid the expensive and time consuming coding of minute descriptions of the target that are typically used in statistical vulnerability analyses.

4. POSSIBILISTIC KILL CRITERIONS

We model our approximate knowledge about the decision function $C(e,t)$ by specifying possibility distributions of the parameters that define the function. This makes $C(e,t)$ a fuzzy function $\tilde{C}(e,t)$. The corresponding decision equation $\tilde{C}(e,t) = 0$ defines in the event space a fuzzy surface (Celmiņš 1987) as the boundary of a set $\tilde{K}$ that contains the events that constitute a kill. (The "kill set" $\tilde{K}$ is a fuzzy set because its boundary is the fuzzy surface $\tilde{C} = 0$.) Let the membership function of the fuzzy boundary be $\mu_{\tilde{C}=0}(e,t)$ and let a corresponding crisp equation $C(e,t) = 0$ define a crisp decision boundary within the core of the fuzzy boundary. Then the possibility that the crisp event $g = (e,t)$ corresponds to a kill is a fuzzy set with the membership function (Celmiņš 1989)

$$\rho_K(e,t) = \rho_{g \in \widetilde{K}}(e,t) = \begin{cases} \mu_{\widetilde{C}=0}(e,t) & \text{if} \quad C(e,t) < 0 \ , \\ 1 & \text{if} \quad C(e,t) \geq 0 \ . \end{cases} \tag{9}$$

The membership function of the necessity that the crisp event g corresponds to a kill is

$$\nu_K(e,t) = \nu_{g \in \widetilde{K}}(e,t) = \begin{cases} 0 & \text{if} \quad C(e,t) < 0 \ , \\ 1 - \mu_{\widetilde{C}=0}(e,t) & \text{if} \quad C(e,t) \geq 0 \ . \end{cases} \tag{10}$$

Figure 5 shows the possibility that a crisp threat parameter t is greater than a fuzzy damage threshold $\widetilde{m}$ (computed with Eq. (9)). The corresponding necessity, computed with Eq. (10), is shown in Figure 6. If the damage threshold is crisp, then both sets are identical and crisp.

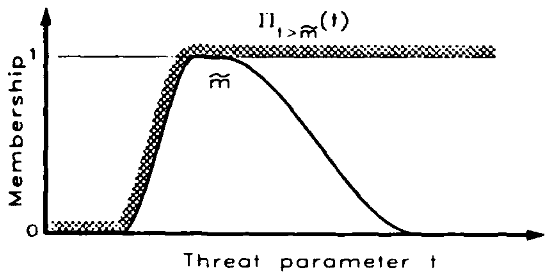

Figure 5. Damage possibility

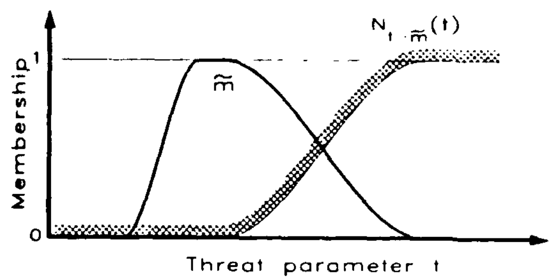

Figure 6. Damage necessity

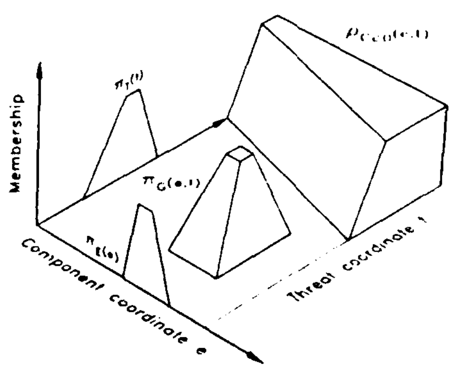

Figure 7. Fuzzy event and fuzzy criterion

Next, we consider fuzzy events $\widetilde{g} = (\widetilde{e},\widetilde{t})$ where the target element, the threat and the decision surface are all fuzzy. Let the membership functions of the target element and threat be $\pi_E(e)$ and $\pi_T(t)$, respectively. The possibility distribution $\pi_G(e,t)$ of the event $\widetilde{g}$ is obtained by a conjunctive combination (Eq. (4)) of these distributions:

$$\pi_G(e,t) = \min \{ \pi_E(e) , \pi_T(t) \} \tag{11}$$

Figure 7 illustrates the result in a two-dimensional event space (see also Figure 2). The pyramid represents the distribution π_G, while $\pi_E(e)$ and $\pi_T(t)$ are projections of the pyramid onto the corresponding planes. We want to determine to what extent the pyramid is inside the kill plateau (or behind a fuzzy boundary of the plateau). The possibility and necessity are two measures that express this extent. The possibility that $\tilde{g}$ corresponds to a kill is measured by the degree of possibility that an element of the set $\tilde{g}$ matches an element of the set $\tilde{K}$. That possibility is computed by (Dubois, Prade, and Testemale 1988)

$$\Pi_K(\tilde{g}) = \Pi_{\tilde{g} \subseteq \tilde{K}} = \sup_{e,t} \min \{ \pi_G(e,t) , \rho_K(e,t) \} \ . \tag{12}$$

The necessity of a match is calculated by

$$\mathrm{N}_K(\tilde{g}) = \mathrm{N}_{\tilde{g} \subseteq \tilde{K}} = 1 - \sup_{e,t} \min \{ \pi_G(e,t) , 1 - \nu_K(e,t) \} \ . \tag{13}$$

5. AGGREGATION

In a crisp environment, the state of a target element is described by the binary state functions $S^{(0)}(e)$ and $S^{(1)}(e,t)$. The value of the state function for the whole target is obtained by aggregation of the state functions of the elements with the help of a deactivation diagram or fault tree (Rapp 1983). In a possibilistic setting, we compute the values of the possibility $\Pi_K^{(1)}$ and necessity $\mathrm{N}_K^{(1)}$ instead of a single state value $S^{(1)}$, and aggregate the elemental possibilities and necessities separately. The formulas for a single element $\tilde{g} = (\tilde{e},\tilde{t})$ are

$$\Pi_K^{(1)}(\tilde{g}) = 1 - S^{(0)}(\tilde{e}) \cdot (1 - \Pi_K(\tilde{g})) \tag{14}$$

and

$$\mathrm{N}_K^{(1)}(\tilde{g}) = 1 - S^{(0)}(\tilde{e}) \cdot (1 - \mathrm{N}_K(\tilde{g})) \ . \tag{15}$$

The aggregation is done as follows.

Let $\tilde{f}$ be a higher level system component consisting of the elementary system components $\tilde{e}_1,...,\tilde{e}_m$. We are seeking the possibility and necessity that the event $\tilde{q} = (\tilde{f},\tilde{t})$ corresponds to a kill of $\tilde{f}$. We do the calculations in two steps. In the first step, we compute transfer values $\Pi_K(\tilde{q})$ and $\mathrm{N}_K(\tilde{q})$ by aggregation of the corresponding elemental values for the events $\tilde{g}_i = (\tilde{e}_i,\tilde{t})$. In the second step, we compute from the transfer values the possibility $\Pi_K^{(1)}(\tilde{q})$ and necessity $\mathrm{N}_K^{(1)}(\tilde{q})$ of the outcome.

The aggregation formulas for the transfer values depend on the logical connection between $\tilde{f}$ and its elements $\tilde{e}_i$ specified, for instance, by a deactivation diagram or fault

tree. If the connection is conjunctive, then the aggregations are

$$\left.\begin{aligned} \Pi_K(\tilde{q}) &= \min_{i=1,\ldots,m} \Pi_K^{(1)}(\tilde{g}_i) \ , \\ N_K(\tilde{q}) &= \min_{i=1,\ldots,m} N_K^{(1)}(\tilde{g}_i) \ . \end{aligned}\right\} \tag{16}$$

If the connection is disjunctive, then the aggregation formulas are

$$\left.\begin{aligned} \Pi_K(\tilde{q}) &= \max_{i=1,\ldots,m} \Pi_K^{(1)}(\tilde{g}_i) \ , \\ N_K(\tilde{q}) &= \max_{i=1,\ldots,m} N_K^{(1)}(\tilde{g}_i) \ . \end{aligned}\right\} \tag{17}$$

To have more flexibility, one can introduce weights in the aggregation process (Dubois, Prade, and Testemale 1988). The weight w_i expresses the relative importance of the element $\tilde{e}_i$ and the decision function $\tilde{C}_i$ for the state of the combined element $\tilde{f}$. A larger weight means that the damage of the element is more important; in the context of pattern matching, a larger weight means that the element has a higher significance. Let the weights be normalized by

$$\max_{i=1,\ldots,m} w_i = 1 \ . \tag{18}$$

The weighted transfer formulas for a conjunction are, in generalization of Eq. (16),

$$\left.\begin{aligned} \Pi_K(\tilde{q}) &= \min_{i=1,\ldots,m} \max \left\{ 1 - w_i \, , \Pi_K^{(1)}(\tilde{g}_i) \right\} \ , \\ N_K(\tilde{q}) &= \min_{i=1,\ldots,m} \max \left\{ 1 - w_i \, , N_K^{(1)}(\tilde{g}_i) \right\} \ . \end{aligned}\right\} \tag{19}$$

The corresponding formulas for weighted disjunction are, in generalization of Eq. (17),

$$\left.\begin{aligned} \Pi_K(\tilde{q}) &= \max_{i=1,\ldots,m} \min \left\{ w_i \, , \Pi_K^{(1)}(\tilde{g}_i) \right\} \ , \\ N_K(\tilde{q}) &= \max_{i=1,\ldots,m} \min \left\{ w_i \, , N_K^{(1)}(\tilde{g}_i) \right\} \ . \end{aligned}\right\} \tag{20}$$

In the second step of the calculation, we compute the possibility and necessity that the combined system component $\tilde{f}$ is killed by the formulas

$$\left.\begin{aligned} \Pi_K^{(1)}(\tilde{q}) &= 1 - S^{(0)}(\tilde{f}) \cdot \left(1 - \Pi_K(\tilde{q}) \right) \ , \\ N_K^{(1)}(\tilde{q}) &= 1 - S^{(0)}(\tilde{f}) \cdot \left(1 - N_K(\tilde{q}) \right) \ . \end{aligned}\right\} \tag{21}$$

6. EXAMPLE

We present as an example the calculation of possibility and necessity of kill of a hypothetical tank. A side view of the tank is shown in Figure 8 where the silhouette is overlaid with a rectangular grid. We compute the possibilistic vulnerability measures for shots at the center of each grid cell and calculate sight averages by summing the cell results and dividing by the number of cells. In this example, there are 1205 cells in the silhouette. The example obviously does not resemble any existing or contemplated weapon system. However, it is an approximate description of a typical tank in the sense that the tank's size, armor, and reactions to the threat have realistic orders of magnitude. Therefore, the example is a valid test problem for the feasibility of possibilistic vulnerability analyses. In particular we tested in this example the sensitivity of the vulnerability indicators (possibility and necessity) to the fuzziness of the input.

The description of the target consisted of its silhouette as shown in Figure 8, information about the armor thickness for various parts of the silhouette (Figure 9), and the outlines of six "elementary" compartments of the target (Figures 10 through 15).

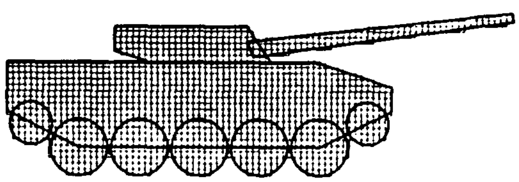

Figure 8. Target silhouette

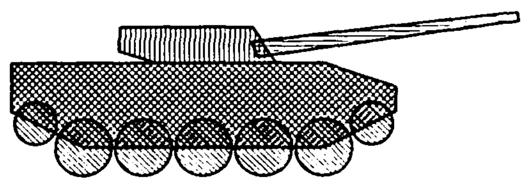

Figure 9. Armor thickness

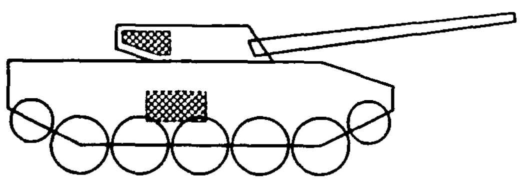

Figure 10. Ammunition compartment

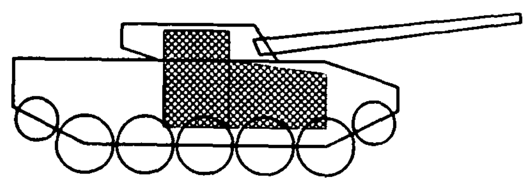

Figure 11. Crew compartment

The calculations were done for three kill categories that were defined in terms of compartment kills as follows. Let the compartments be denoted by A — ammunition, C — crew, E — engine, F — fuel, G — driving gear and wheels, and W — weapon. A

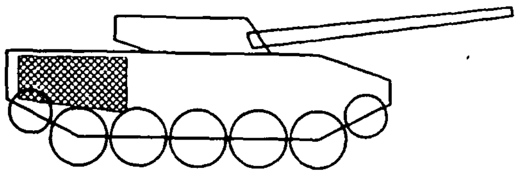

Figure 12. Engine compartment

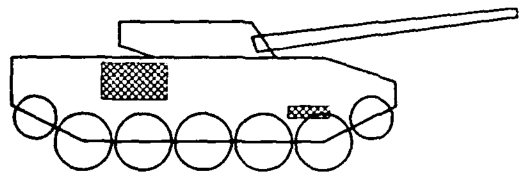

Figure 13. Fuel compartment

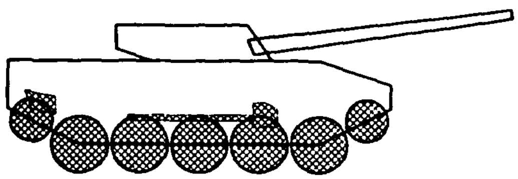

Figure 14. Driving gear compartment

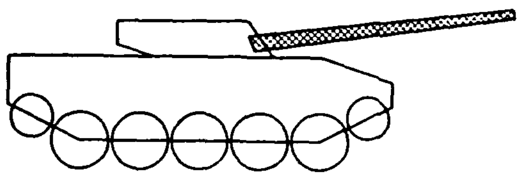

Figure 15. Weapon compartment

"mobility kill" was defined by

$$\mathbf{M}_K \longleftrightarrow C_K \vee E_K \vee G_K \tag{22}$$

meaning that a mobility kill occurs if either the crew or the engine or the driving gear is killed. A firepower kill was defined by

$$\mathbf{F}_K \longleftrightarrow C_K \vee W_K \tag{23}$$

and a catastrophic kill was defined by

$$\mathbf{K}_K \longleftrightarrow A_K \vee F_K \vee [\, C_K \wedge (\, E_K \vee G_K \,) \wedge W_K \,] \; . \tag{24}$$

Aggregation was done for individual shots, and sight averages were computed as indicated.

The threat was characterized by two hypothetical curves (see Figures 16 and 17) that provide the residual mass and energy of the projectile as functions of armor thickness. (The residual mass and energy are those of the round after penetration of the armor. In the present example, the round can penetrate armors with thicknesses up to 0.3 m.) We assumed that the estimates of these residuals were not exactly known but given as fuzzy curves, indicated by the shaded areas in the figures. Each figure also contains three crisp curves. The center curves are the cores of the fuzzy residual estimates, and the top and bottom curves outline the supports of the estimates.

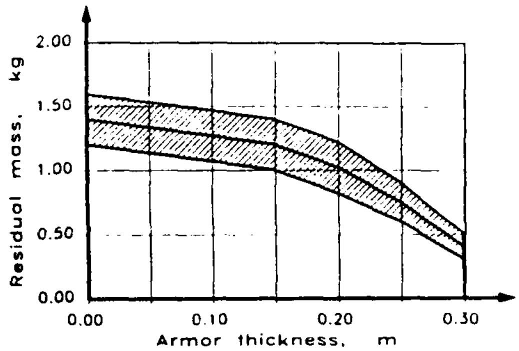

Figure 16. Residual Mass.

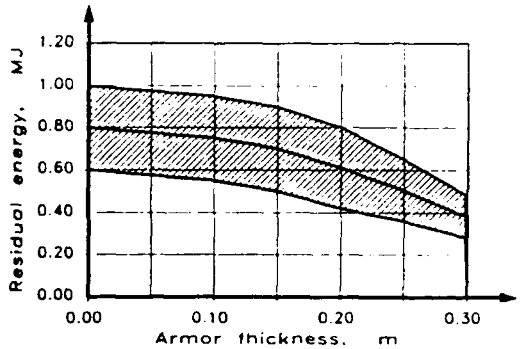

Figure 17. Residual Energy.

The response of each element (compartment) was represented by a fuzzy decision curve in the (residual mass, residual energy)-plane. Figure 18 shows the decision curve for the ammunition compartment. The ammunition is assumed to be killed if the reference point is above the decision curve, for instance, in the upper right hand corner of the figure. The decision curves for other compartments are shown in Figures 19 through 23.

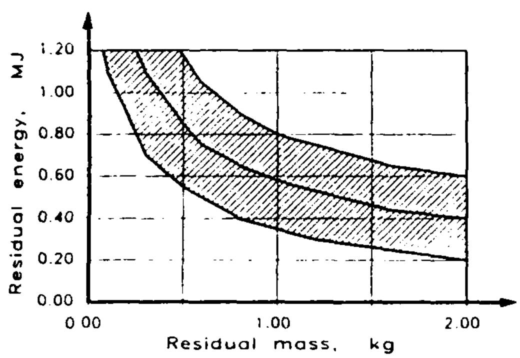

Figure 18. Criterion for ammunition

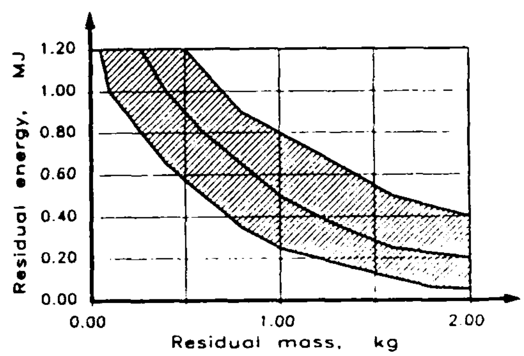

Figure 19. Criterion for crew

The calculation of the possibility and necessity of kill was done for each of the 1205 shotlines through the centers of the grid cells in Figure 8 as follows. First, the armor thickness was determined from Figure 9 and trajectory data. This determined with the aid of the curves shown in Figures 16 and 17 the corresponding (fuzzy) residual mass and energy, i.e., a (fuzzy) event point in the mass,energy-plane. We then determined which compartments were affected by the shot (see Figures 10 through 15) and used the proper decision curves (see Figures 18 through 23) to compute the possibility and necessity that the event corresponded to a kill of a compartment, i.e.,

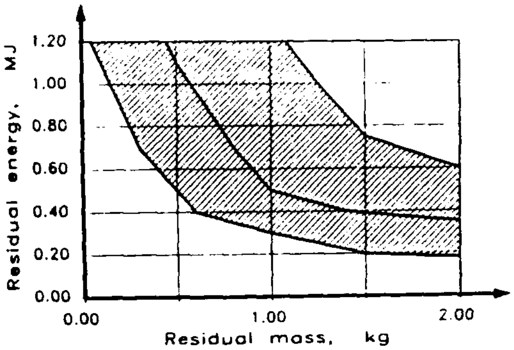

Figure 20. Criterion for engine

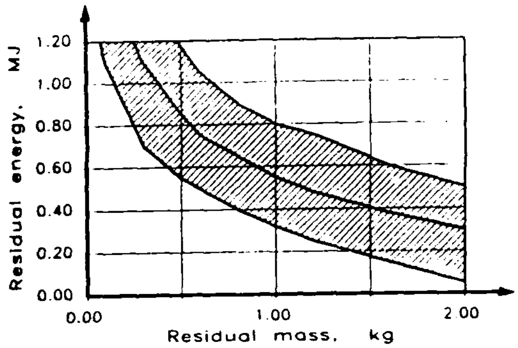

Figure 21. Criterion for fuel

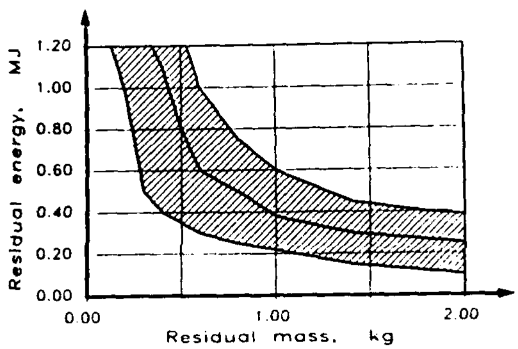

Figure 22. Criterion for driving gear

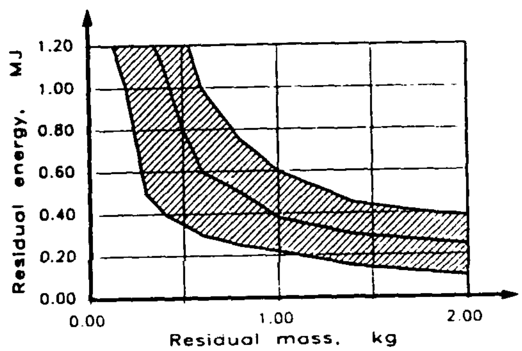

Figure 23. Criterion for weapon

that the event point is within the kill space. Finally, we aggregated with Eqs. (22), (23), and (24) (using the rules (4) and (5)) the compartment values to obtain the mobility, firepower, and catastrophic kill possibilities and necessities for the tank. After completed calculations for all shotlines, sight averages were computed by unweighted averaging over the silhouette.

We performed four calculations. First, we assumed that all input is crisp, that is, all fuzzy input curves were replaced by the corresponding core curves. In that case, every compartment is either intact or killed after a shot. In the remaining calculations, the input is fuzzy and we have available after a shot for each compartment the values of possibility and necessity of kill. In the next two cases, we assumed that only the threat characterization and only the decision curves were fuzzy, respectively. Finally, we assumed that all input curves were fuzzy. The results of the four calculations are listed in Table 1 and displayed in Figures 23, 24, and 25.

We explain the display by considering the firepower kill. In the all-crisp case, the possibility and necessity both equal 0.229. The value is listed in Table 1 in the column "Core" (or "Fuzzy: None") and shown in Figure 24 above the label "Crisp". When only the target is fuzzy, we obtain $\Pi_K = 0.231$ and $N_K = 0.164$. The values are listed in Table 1 in the columns "Fuzzy: Target" and shown in Figure 24 by a bar above the label "Fuzzy target". The top of the bar shows the possibility value and the bottom of the bar corresponds to the necessity value. If only the threat is fuzzy then $\Pi_K = 0.230$ and $N_K = 0.112$. Table 1 lists these values in the columns "Fuzzy: Threat" and Figure 24 shows the corresponding bar above the label "Fuzzy threat". Finally, in the all-fuzzy case we have $\Pi_K = 0.245$ and $N_K = 0.102$. The values are listed in Table 1 in the columns "Fuzzy: All" and shown in Figure 24 by the bar above the label "Fuzzy". The listings and displays of the mobility and catastrophic kill cases are arranged in the same manner.

The table and the figures show that the spreads between possibility and necessity increase as more of the input becomes fuzzy. Note, however, that the spread of the result is not excessively large in spite of liberal assignments of spreads to the input. Hence the possibilistic vulnerability measures are reasonable and useful indicators of vulnerability at least in the present example.

Table 1. Possibilities and Necessities for Different Kill Categories.

	Possibility			Core	Necessity		
Fuzzy:	All	Threat	Target	None	Target	Threat	All
Firepower	0.254	0.230	0.231	0.229	0.164	0.112	0.102
Mobility	0.605	0.563	0.577	0.550	0.420	0.349	0.328
Catastr.	0.073	0.065	0.065	0.065	0.021	0.014	0.011

7. SUMMARY AND CONCLUSIONS

We have described how possibility theory and pattern matching can be used to derive new measures of vulnerability of approximately described targets. The new measures are the possibility and necessity of killing a target. They may replace the common probability of kill if the target and threat descriptions are approximate but not stochastic. Such descriptions are typical for hypothetical and generic weapons that are investigated in the early phases of weapon development. The new measures express the confidence that such described targets will or will not be killed. The new vulnerability measures can be used to provide quick answers to questions about the relative merit of

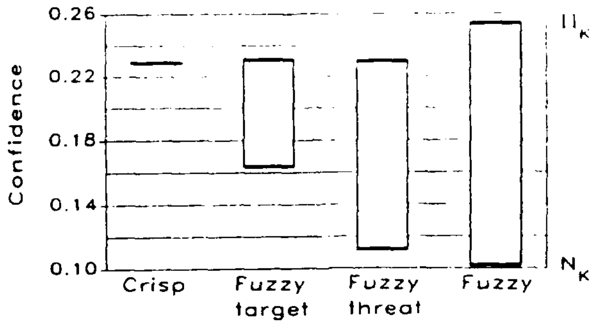

Figure 24. Firepower kill

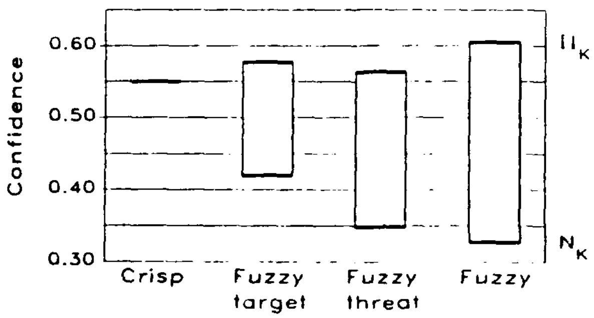

Figure 25. Mobility kill

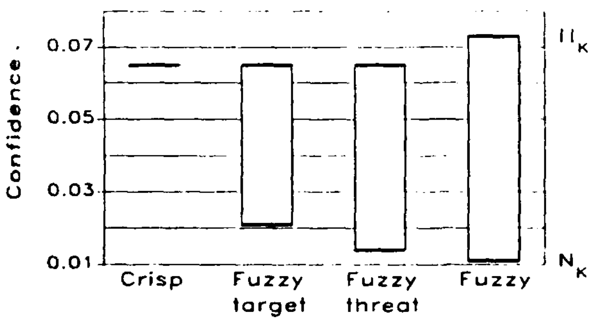

Figure 26. Catastrophic kill

modifications of hypothetical subsystems. Probabilistic approaches are problematic in such cases because of the difficulty to define a meaningful population to which statistical

methods can be applied.

The computations of the possibility and necessity of kill are based on the same terminal ballistics and fault tree model concepts as common probabilistic calculations, except that the models and inputs are assumed to be fuzzy where appropriate. This ensures consistency between the different vulnerability measures; if the models and inputs are crisp (exact), then possibilistic and probabilistic analyses yield identical results in the sense that in both cases the same components will be determined as killed. The advantages of a possibilistic calculation are that it allows a simpler problem specification when details are not known or not needed. This translates into a faster setup time for the problem.

One purpose of the example in this report was to test the sensitivity of possibilistic vulnerability measures to the fuzziness of the input in a typical calculation. The numerical results demonstrate that one can expect meaningful results even with a very liberal assignment of uncertainties.

We conclude that possibilistic vulnerability measures offer a viable alternative to traditional measures in cases when non-stochastic inaccuracies must be taken into account.

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